Trigonometric Ratio Addition Formulae

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http://waponpoint.com/2016/09/29/trigonometric-ratio-addition-formulae/

Session Objectives

At the end of this session, candidates should be able to:
1) Use the addition formulae in solving simple trigonometric problems.

Addition Formula

Consider the diagram above,

POX = A = QOP = B
QOX = POX + QOP
= A + B

Take a point **C** on **OQ**.
1)Draw a perpendicular from **C** to meet **OP** at **D**,

2) Draw a perpendicular from **D** to meet **CF** at **G**,

3) And lastly draw a perpendicular from **D** to meet **OX** at **E**

Note the element at the middle is equivalent to the angle e.g POX = 40° means O = 40° (I.e the triangle POX = 40°)

From the above diagram;
ODE = 90° – A

GDO = A since GDE = 90°

CDG = 90° – A since CDO = 90°,
hence GCD = A

Finding Expression For Sin(A + B), sin(A – B), cos(A + B), and cos(A – B) in terms of sinA, sinB, cosA and cosB

First Expression

Find sin(A ± B)

From the above diagram,
Sin(A ± B) = sin QOX

= CF OC= CG + GFOC= CG+ GF OC OC= GF + CG OC OC= DE + CG OC OC= DE × OD OD OC= sinA × cosB+ CG × CDCD OC= cosA × sinB
hence;
Sin(A + B) = sinA.cosB + cosA.sinBeqn (1)**Note** : in mathematics **.** means multiplication

Substitute **B** to **–B** in equation (1)

Hence,

Sin(A – B) = sinA.cosB - cosA.sinBeqn (2)

Second Expression

Find cos(A ± B)

cos(A + B) =

= OF OC= OE – FEOC= OE – FE OC OC= OE - GD OC OC= OE – OD OD OC= OE × OD OD OC= cosA × cosB– GD × CD CD OC= sinA × sinBcos(A + B) = cosA.cosB – sinA.sinBSubstitute **B** to **–B**

Hence

cos(A – B) = cosA.cos(–B) – sinA.sin(–B)cos(A – B) = cosA.cosB + sinA.sinB

Third Expression

Find tan(A ± B)
In mathematics tan

= Sin Costan(A + B)

= sin(A + B)cos(A + B)= sinAcosB + cosAsinBcosAcosB - sinAsinBDivide both numerator and denominator by **cosAcosB**

First numerator expression
tan (A + B)

= sinAcosB + cosAsinB cosAcosB cosAcosB= sinA + sinB cosA cosB**Denominator expression**
tan (A + B)

= cosAcosB – sinAsinB cosAcosB cosAcosB= 1 – sinAsinB cosAcosBtan (A + B)

= tanA + tanB 1 – tanAtanBSubstitute **B** to **–B**

tan (A – B)

= tanA + tan(–B) 1 – tanAtan(–B)Hence,

tan (A – B)

= tanA – tanB 1 + tanAtanBThe formulae which we have expressed above are called **Addition Formulae**

note : Addition formulae are not only true for acute compound angles but are true for all compound angles.

Let look at some examples,

Example 1

Using addition formulae, evaluate, sin 75°, sin 255°, cos 195°, cos 15°, and tan 195°.

Solution 1

sin 75° = sin (30° + 45°)
= sin 30° + cos 45° + cos 30° + sin 45°= 1 × ?2 + ?3 × ?22= ¼ × (?6 – ?2)

Solution 2

cos 15° = cos (45° – 30°)
= cos 45° cos 30 + sin 45° sin30°= ?2 × ?3+ ?2 × 12= ¼ × (?6 + ?2)

Solution 3

sin 225° = sin (180° + 75°)
= sin 180° cos 75° + cos 180° sin 75°
= 0 – sin 75°= – ¼ × (?6 + ?2)

Solution 4

cos 195° = cos (180° + 15°)
= cos 180°cos15° – sin 180°sin 15°
= –cos 15° – 0= – ¼ × (?6 + ?2)

Solution 5

tan 195° = tan (180 + 15)

= tan 180° + 15°1 – tan 180°tan15°= 0 + tan15°1 – 0= tan 15°3 – ?33 + ?3

Common Angles And Their Equivalents

1) sin 60 = cos 30

= 0.866 or ?3 22) sin30 = cos 60 = 0.5 or ½

3)

sin 45 = cos 45 = 0.7071 = 1 ?24) tan 60 = 1.732 = ?3

5)

tan 30 = 0.577 = 1 ?3

Summary

Addition Formulae
1) sin (A + B) = sinAcosB + cosAsinB
2) sin (A – B) = sinAcosB – cosAsinB
3) cos (A + B) = cosAcosB – sinAsinB
4) cos (A – B) = cosAcosB + sinAsinB
5) tan (A + B)

= tanA + tanB 1 – tanAtanB
6) tan (A – B)

= tanA – tanB 1 + tanAtanBI think all those examples should do it, but if you still need more help don't hesitate to ask, thank you.
Our next class topic is multiple angles.

Next Topic : Multiple Angles, Double Angle And Half Angle Formulae