Mathematics : Product Formulae

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http://waponpoint.com/2016/10/12/mathematics-product-formulae/

Session Objectives

At the end of this chapter, candidates should be able to use the product formulae to solve trigonometric problems.

If you have any question regarding our last Class kindly don't hesitate to ask

Product Formulae

First Expression

Recall that from the addition formulae,
sin (A + B) = sinAcosB + cosAsinB
And
sin (A ? B) = sinAcosB ? cosAsinB
Adding the two expressions together will give us;

sin (A + B) = sinAcosB + cosAsinB
+
sin (A ? B) = sinAcosB ? cosAsinB

sin (A + B) + sin (A ? B) = sinAcosB + cosAsinB + sinAcosB ? cosAsinB

sin (A + B) + sin (A ? B) = 2sinAcosB

Second Expression

sin (A + B) = sinAcosB + cosAsinB
And
sin (A ? B) = sinAcosB ? cosAsinB
Subtracting the two expressions will give us;

sin (A + B) = sinAcosB + cosAsinB
?
sin (A ? B) = sinAcosB ? cosAsinB

sin (A + B) ? sin (A ? B) = sinAcosB + cosAsinB ? (sinAcosB ? cosAsinB) sin (A + B) ? sin (A ? B) = sinAcosB + cosAsinB ? sinAcosB + cosAsinB

sin (A + B) ? sin (A ? B) = 2cosAsinB

Let
X = 2sinAcosB,
Y = 2cosAsinB
P = sin (A + B)
Q = sin (A ? B)

Hence,
X = P + Q and,
Y = P ? Q
X + Y = P + Q + P ? Q
X + Y = 2P
Make **P** the subject of the formula;
X + Y = 2Ptop?

P = X + Y 2
X = P + Q
Y = P ? Q
X ? Y = P + Q ? (P ? Q)
X ? Y = P + Q ? P + Q
X ? Y = 2Q
Make **Q** the subject of the formula;

Q = X ? Y 2Recall that from the first expression;
sin (A + B) + sin (A ? B) = 2sinAcosB

And

Q = X ? Y 2P = X + Y 2then,
sinX + sin Y = ½(2sin P + Qcos P ? Q)

Recall that from the second expression;
sin (A + B) ? sin (A ? B) = 2cosAsinB top?

And

Q = X ? Y 2P = X + Y 2then,
sinX ? sin Y = ½(2cos P + Qsin P ? Q)

Also,
cos (A + B) = cosAcosB ? sinAsinB
And,
cos (A ? B) = cosAcosB + sinAsinB

If we put X = P + Q, and Y = P ? Q
then,

Q = X ? Y 2P= X + Y 2top?
Hence,
cos X + cos Y = ½(2cos P + Qcos P ? Q)

cos X ? cos Y = ½(2sin P + Q sin P ? Q)

All the above formulae that we just expressed are called **Product Formulae**

Now let look at 2 or more examples,

Examples Of Product Of Two Trigonometric Ratio

Example 1

Express sin 4x + sin 2x and sin 8x ? sin 2x as product of two trigonometric ratios.

Solution 1

To solve this question, you don't need any calculations just follow the expressions that will have analyse above and input the parameters.top?

sin 4x + sin 2x = ½(2sin 4x + 2x cos 4x ? 2x)
= 2sin 3x cos x

Solution 2

Also use the same procedure

sin 8x ? sin 2x = ½ (2cos8x + 2x sin 8x ? 2x)
= 2cos 5x sin 3x

Example 2

Express cos 6x + cos 4x and cos 4x ? cos 2x as product of two trigonometric ratios.

Solution 3

cos 6x + cos 4x = ½(2cos 6x + 4xcos 6x ? 4x)
= 2cos 5x cos x

Solution 4

cos 4x ? cos 2x = ½(?2sin 4x + 2xsin 4x ? 2x)
?2sin 3x sin x

Examples Of Sum Of Two Trigonometric Ratio

Example 1

Express sin 5xcos 3x as a sum of two trigonometric ratio.[top?]

Solution

Recall,

sinX + sinY = ½(2sin X + Ycos X ? Y)

Agree that

½[sinX + sinY] = ½(sin X + Ycos X ? Y)

Put

5x = X + Y 2
And

3x = X ? Y 2
then cross multiply,

X + Y = 5x × 2
X + Y = 10x....... eqn (1)

X ? Y = 3x × 2
X ? Y = 6x........ eqn (2)

Solving the two equation simultaneously
X + Y = 10x
X ? Y = 6x[top?]

Making **X** the subject of the formula in equation 2
X ? Y = 6x
X = 6x + Y

Put **X = 6x + Y** into equation 1
X + Y = 10x
(6x + Y) + Y = 10x
6x + 2Y = 10x

2Y = 10x ? 6x
2Y = 4x
Y = 2x

Put **Y = 2x** into equation 2
X ? Y = 6x
X ? (2x) = 6x
X ? 2x = 6x
X = 6x + 2x
X = 8x

So if X = 8x and Y = 2x then,
sin 5xcos 3x = ½[sin 8x + sin 2x]

Example 2 Express cos 7xsin 5x as a sum of two trigonometric ratio.

Solution

Similar to the first example,
cos 7xsin 5x = ½[sinX ? sinY]top?

Put

7x = X + Y 2
And

5x = X ? Y 2
then cross multiply,

X + Y = 7x × 2
X + Y = 14x...... eqn (1)

X ? Y = 5x × 2
X ? Y = 10x .... eqn (2)

Solving the two equations simultaneously,
X + Y = 14x
X ? Y = 10x

Making X the subject of the formula in equation 1
X + Y = 14x
X = 14x ? Y

Put **X = 14x ? Y** into equation 2
X ? Y = 10x

(14x ? Y) ? Y = 10x
14x ? Y ? Y = 10x
14x ? 2 Y = 10x
?2Y = 10x ? 14x
?2Y = ? 4x
Y = 2xtop?

Put **Y = 2x** into equation 1
X + Y = 14x
X + (2x) = 14x
X + 2x = 14x
X = 14x ? 2x
X = 12x

So if X = 12x and Y = 2x then,

cos 7xsin 5x = ½[sin 12x ? sin2x]

Example 3

Express cos 9xcos 3x as a sum of two trigonometric ratio.

Solution

Same procedure,
cos 9xcos 3x = ½[cosX + cosY]

Put

9x = X + Y 2
And

3x = X ? Y 2
then cross multiply,

X + Y = 9x × 2
X + Y = 14x....... eqn (1)

X ? Y = 3x × 2
X ? Y = 6x........ eqn (2)

Solving the two equations simultaneously

X + Y = 14x
X ? Y = 6x

Make **X or Y** the subject of the formula in any of the equation. (Your choice)
Making **X** the subject of the formula in equation 2top?

X ? Y = 6x
X = 6x + Y

Put **X = 6x + Y** into equation 1
X + Y = 14x
(6x + Y) + Y = 14x
6x + Y + Y = 14x
6x + 2Y = 14x
2Y = 14x ? 6x
2Y = 12x
Y = 6x

Put **Y = 6x** into equation 2
X ? Y = 6x
X ? (6x) = 6x
X ? 6x = 6x
X = 6x + 6x
X = 12x

So if X = 12x and Y = 6x then,

cos 9x cos 3x = ½[cos 12x + cos 6x]

Question Of The Day

Express sin 3xsinx as a sum of two trigonometric ratio? Submit your answer through the comment box.

Jokes Of The Day

I can't stop laughing when I saw this picture, lol, it is true tho...

Is this true?top?

Feel free to ask our tutors any question via the comment box or ask question page and I will be happy to answer your question and take the session again if necessary, your comment notifications is like a bank alert to us so don't forget to write something.
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